VECTORS IN R^n, OPERATIONS, SCALAR PRODUCT

Mathematics I — Exercises for Land Resources Management

Academic Year 2025/2026

Prof. UPP Agnieszka Łacka, PhD, DSc

Department of Mathematical and Statistical Methods

Poznań University of Life Sciences

1. Given points $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$. Find the components and lengths of the vectors: $\vec{\mathbf{AB}}$, $\vec{\mathbf{AC}}$, $\vec{\mathbf{CB}}$. Draw a diagram:

   (a) $\mathbf{A} = (-1, 2)$, $\mathbf{B} = (1, 0)$, $\mathbf{C} = (2, 2)$

   (b) $\mathbf{A} = (1, -4)$, $\mathbf{B} = (5, 3)$, $\mathbf{C} = (2, 5)$

   (c) $\mathbf{A} = (-4, 6)$, $\mathbf{B} = (-3, -1)$, $\mathbf{C} = (6, -4)$

   (d) $\mathbf{A} = (7, -3)$, $\mathbf{B} = (-2, 4)$, $\mathbf{C} = (-4, 6)$

   (e) $\mathbf{A} = (2, 3)$, $\mathbf{B} = (-6, 0)$, $\mathbf{C} = (3, 2)$

   (f) $\mathbf{A} = (-1, 3)$, $\mathbf{B} = (1, 4)$, $\mathbf{C} = (-2, 2)$

   (g) $\mathbf{A} = (3, 2)$, $\mathbf{B} = (1, 3)$, $\mathbf{C} = (1, 2)$

   (h) $\mathbf{A} = (-2, 2)$, $\mathbf{B} = (3, 0)$, $\mathbf{C} = (0, 2)$

2. Given vectors $\mathbf{u}$ and $\mathbf{v}$. Find the vectors $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, $2\mathbf{u}$, $-\frac{1}{3}\mathbf{v}$, $3\mathbf{u}-2\mathbf{v}$, $\frac{1}{2}\mathbf{u}-\frac{2}{3}\mathbf{v}$, $\frac{1}{3}\mathbf{u}-\frac{2}{3}\mathbf{v}$ and compute their lengths:

   (a) $\mathbf{u}^T = [2, -3]$, $\mathbf{v}^T = [1, 4]$

   (b) $\mathbf{u} = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

   (c) $\mathbf{u}^T = [-1, 2, 1]$, $\mathbf{v}^T = [1, 0, 2]$

   (d) $\mathbf{u} = \begin{bmatrix} 7 \\ -2 \\ -2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 5 \\ -4 \end{bmatrix}$

   (e) $\mathbf{u} = 2\begin{bmatrix} 6 \\ 0 \end{bmatrix}$, $\mathbf{v} = -\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

   (f) $\mathbf{u} = -3\mathbf{i} + \mathbf{j}$, $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j}$

   (g) $\mathbf{u} = 8\mathbf{j}$, $\mathbf{v} = (-3)(-2\mathbf{i} + \mathbf{j})$

   (h) $\mathbf{u}^T = -2[0, 7, 2]$, $\mathbf{v}^T = 5[1, 2, 2]$

   (i) $\mathbf{u} = 3\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v} = -2\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

   (j) $\mathbf{u} = -(4\mathbf{i} - \mathbf{j})$, $\mathbf{v} = 2(\mathbf{i} - 3\mathbf{j})$

   (k) $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -6\mathbf{i} + 4\mathbf{k}$

3. Compute the scalar product $\mathbf{u} \circ \mathbf{v}$ for the pairs of vectors from Task 3.

4. Compute the cosine of the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$ (for the pairs of vectors from Task 3).

5. Given points $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R} = (x, y, z)$, find values x, y and z such that the vector $\vec{\mathbf{PR}}$ is orthogonal to the vector $\vec{\mathbf{PQ}}$:

   (a) $\mathbf{P} = (8, -3, 5)$, $\mathbf{Q} = (6, 1, 7)$

   (b) $\mathbf{P} = (2, 1, 5)$, $\mathbf{Q} = (8, 4, 6)$

   (c) $\mathbf{P} = (1, 0, 0)$, $\mathbf{Q} = (0, 1, 0)$

   (d) $\mathbf{P} = (3, -5, 5)$, $\mathbf{Q} = (2, 1, 6)$

   (e) $\mathbf{P} = (4, 0, -1)$, $\mathbf{Q} = (2, 4, 2)$

   (f) $\mathbf{P} = (0, 1, 4)$, $\mathbf{Q} = (5, 2, 1)$

   (g) $\mathbf{P} = (1, 2, 3)$, $\mathbf{Q} = (2, 2, 2)$