

MATRIX TYPES AND MATRIX OPERATIONS

Mathematics I — Exercises for Land Resources Management

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1. Compute $\mathbf{A} + \mathbf{B}$, $\mathbf{A} - \mathbf{B}$, $\mathbf{A}^T$, $3\mathbf{B}$, $\mathbf{B}^T - \mathbf{C}^T$, $(\mathbf{B} + \mathbf{C})^T$, $\mathbf{B} - \mathbf{C}$, $\mathbf{A}^T - 2\mathbf{B}^T + \frac{1}{3}\mathbf{C}^T$, if:

$$(a) \mathbf{A} = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 1 & -4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & -1 & 2 \\ 3 & 1 & 4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & -3 & 9 \\ 3 & 18 & -6 \end{bmatrix}$$

$$(b) \mathbf{A} = \begin{bmatrix} 1 & -1 & 5 & 1 \\ 2 & 5 & -2 & 5 \\ 2 & 1 & -4 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 4 & -2 & 3 & 1 \\ 2 & 5 & -2 & 2 \\ 1 & 4 & -4 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & -3 & -3 & 12 \\ 3 & 3 & -6 & 3 \\ 9 & 9 & 0 & 6 \end{bmatrix}$$

$$(c) \mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & -2 \\ 2 & 1 & -7 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & -3 & 5 \\ 3 & 0 & 8 \\ 5 & 2 & -4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 3 & -9 \\ 3 & -12 & 6 \\ 3 & 18 & -3 \end{bmatrix}$$

$$(d) \mathbf{A} = \begin{bmatrix} 0 & -3 & 7 \\ 2 & 6 & -4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -1 & -3 & 2 \\ 3 & 6 & 4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & -3 & -9 \\ -3 & -18 & -6 \end{bmatrix}$$

$$(e) \mathbf{A} = \begin{bmatrix} 11 & -13 & 6 & 9 \\ 4 & 15 & -5 & 0 \\ 2 & 12 & -14 & 11 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & -2 & 3 & 1 \\ 2 & 0 & -12 & 2 \\ 1 & 4 & 0 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & -3 & -3 & 6 \\ 3 & 0 & -6 & 0 \\ -9 & 9 & 0 & 6 \end{bmatrix}$$

$$(f) \mathbf{A} = \begin{bmatrix} -1 & 3 & 0 \\ 4 & 0 & -7 \\ 0 & 6 & -7 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 13 & 5 \\ 0 & 0 & 8 \\ 1 & 2 & -4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & -9 & -9 \\ 3 & -3 & 6 \\ 0 & 3 & -9 \end{bmatrix}$$

2. Compute, if possible: $\mathbf{A}^2$, $\mathbf{B}\mathbf{B}^T$, $\mathbf{B}\mathbf{C}$, $\mathbf{B}\mathbf{D}^T$, $\mathbf{C}\mathbf{D}$, $\mathbf{D}\mathbf{B}$

$$(a) \mathbf{A} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & -1 & -5 \\ -2 & 1 & 4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & 2 & 3 & -2 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 3 & -2 \\ -1 & 0 \\ 2 & 1 \\ 0 & -3 \end{bmatrix}$$

$$(b) \mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 6 \\ 5 & 1 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & -2 & 3 & 4 \\ -1 & 1 & 1 & -1 \\ 1 & 0 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 2 & -1 & 3 & -1 & 0 \\ -1 & 1 & 0 & -1 & 3 \\ 1 & 1 & 0 & 1 & 2 \\ 0 & 2 & 3 & -2 & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 3 & -2 & 1 & 1 \\ 4 & -1 & 0 & 1 \\ 3 & 0 & 1 & 1 \\ -2 & -1 & 1 & 1 \\ 3 & -2 & 1 & 1 \end{bmatrix}$$

$$(c) \mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & -3 & -5 \\ -2 & 5 & 4 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} -4 & 1 & 0 & -1 \\ 1 & 6 & 0 & 1 \\ 0 & 2 & 3 & 2 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 3 & -2 \\ -2 & 6 \\ 2 & 1 \\ 4 & -3 \end{bmatrix}$$

$$(d) \mathbf{A} = \begin{bmatrix} 5 & 2 & 3 \\ 3 & -1 & 6 \\ 5 & 1 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 & 4 \\ -1 & 0 & 1 & -1 \\ 2 & 0 & 1 & 0 \\ 4 & 1 & 1 & 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0 & -1 & 3 & -1 & 0 \\ 1 & 4 & 0 & -1 & -3 \\ -2 & 2 & 3 & 1 & 2 \\ 0 & 2 & 2 & 2 & -1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 2 & -3 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 3 & 0 & 1 & 1 \\ -2 & -2 & 1 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$$

3. Solve the matrix equation for $\mathbf{A} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 1 & 0 & 5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$:

(a) $2\mathbf{A} + \mathbf{X} = \mathbf{B}$

(b) $\mathbf{A} = \mathbf{X} - 3\mathbf{I}$

(c) $\mathbf{B} - 2\mathbf{X} = 5\mathbf{A}$

4. Determine the rank of the matrix:

$$(a) \mathbf{A} = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 1 & -4 \end{bmatrix}$$

$$(b) \mathbf{A} = \begin{bmatrix} 1 & -1 & 5 & 1 \\ 2 & 5 & -2 & 5 \\ 2 & 1 & -4 & 1 \end{bmatrix}$$

$$(c) \mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & -2 \\ 2 & 1 & -7 \end{bmatrix}$$

$$(d) \mathbf{A} = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 1 & -4 \\ 1 & 0 & 3 \\ 1 & 2 & 5 \\ 1 & 1 & -2 \end{bmatrix}$$

$$(e) \mathbf{A} = \begin{bmatrix} 2 & -1 & 3 & -1 & 0 \\ -1 & 1 & 0 & -1 & 3 \\ 1 & 1 & 0 & 1 & 2 \\ 0 & 2 & 3 & -2 & 1 \end{bmatrix}$$

$$(f) \mathbf{A} = \begin{bmatrix} 11 & -13 & 6 & 9 \\ 4 & 15 & -5 & 0 \\ 2 & 12 & -14 & 11 \end{bmatrix}$$

$$(g) \mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 8 & 10 \\ 1 & 0 & -1 & -2 & -3 \\ 2 & 0 & -2 & -4 & -6 \end{bmatrix}$$