Determinant and Inverse Matrix

Mathematics I — Exercises for Land Resources Management

Academic Year 2025/2026

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1. Calculate the determinants:

(a) $\begin{vmatrix} 3 & -4 \\ -1 & 8 \end{vmatrix}$ (b) $\begin{vmatrix} -3 & 4 \\ 6 & -12 \end{vmatrix}$ (c) $\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix}$

(d) $\begin{vmatrix} 1 & -2 \\ -1 & 2 \end{vmatrix}$ (e) $\begin{vmatrix} -4 & 4 \\ -7 & 12 \end{vmatrix}$ (f) $\begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix}$

2. Calculate the determinant using Sarrus' rule

(a) $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ -1 & 1 & 1 \end{vmatrix}$ (b) $\begin{vmatrix} 8 & 5 & -16 \\ 6 & 2 & -12 \\ 1 & -7 & -2 \end{vmatrix}$ (c) $\begin{vmatrix} 3 & -1 & -3 \\ 0 & 4 & 2 \\ 6 & -1 & 5 \end{vmatrix}$

(d) $\begin{vmatrix} 1 & -1 & 2 \\ -1 & 3 & -1 \\ 2 & 3 & 2 \end{vmatrix}$ (e) $\begin{vmatrix} 2 & -1 & 3 \\ 7 & 2 & -4 \\ 1 & -3 & 1 \end{vmatrix}$ (f) $\begin{vmatrix} 1 & a & -b \\ -a & a+b & a-b \\ b & a-b & a+b \end{vmatrix}$

3. Using the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 2 & 1 & 1 \end{bmatrix}$$

(a) Find the minors and cofactors for each element of the second row,

(b) Find the minors and cofactors for each element of the third row,

(c) Find the minors and cofactors for each element of the first column,

(d) Find the minors and cofactors for each element of the third column,

(e) Compute $|\mathbf{A}|$ by expansion along the second row,

(f) Compute $|\mathbf{A}|$ by expansion along the third row,

(g) Compute $|\mathbf{A}|$ by expansion along the first column,

(h) Compute $|\mathbf{A}|$ by expansion along the third column.

4. Compute the determinant using Laplace's expansion

(a) $\begin{vmatrix} 1 & -1 & 2 & 3 \\ 0 & 5 & 7 & 1 \\ -2 & 2 & -4 & -6 \\ 1 & 0 & -1 & 0 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 \\ 1 & 1 & -1 & 3 \\ 0 & 5 & 1 & 0 \end{vmatrix}$ (c) $\begin{vmatrix} 1 & 2 & 3 & 1 \\ -1 & 0 & 0 & 2 \\ 0 & -1 & 2 & -1 \\ 2 & 0 & -1 & 1 \end{vmatrix}$

(d) $\begin{vmatrix} 1 & -1 & 2 & 3 \\ 0 & 5 & 7 & 1 \\ -2 & 1 & 0 & 0 \\ 1 & 1 & -1 & -1 \end{vmatrix}$ (e) $\begin{vmatrix} 0 & 0 & -2 & 3 \\ 1 & 0 & 1 & 2 \\ -1 & 1 & 2 & 1 \\ 0 & 2 & -3 & 0 \end{vmatrix}$ (f) $\begin{vmatrix} 1 & 2 & 0 & -1 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & -1 \\ 2 & 0 & 0 & -3 \end{vmatrix}$

5. Compute the determinant using elementary row and column operations (reducing to upper or lower triangular form)

(a) $\begin{vmatrix} 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 2 & 1 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{vmatrix}$

6. For what values of x is the determinant of matrix $\mathbf{A}$ equal to zero:

(a) $\mathbf{A} = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & x & 2 & 1 \\ -1 & 0 & 1 & x \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (b) $\mathbf{A} = \begin{bmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ (c) $\mathbf{A} = \begin{bmatrix} x & 2 & 1 & 0 \\ 0 & x & 1 & 2 \\ 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

7. Find the inverse matrix using both the **cofactor method** and the **Gauss–Jordan elimination method**:

(a) $\begin{bmatrix} 3 & -1 \\ -3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -1 & 0 \\ 3 & 0 & 2 \\ -1 & 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 5 & 0 \\ 1 & 2 & 1 \\ 3 & 7 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} -1 & -1 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 2 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 3 & 2 \\ -2 & -5 & -7 \\ -1 & -2 & -4 \end{bmatrix}$

(g) $\begin{bmatrix} -2 & 2 & 1 \\ -3 & 1 & 1 \\ 1 & -4 & -1 \end{bmatrix}$ (h) $\begin{bmatrix} -1 & 4 & 5 & 2 \\ 0 & 0 & 0 & -1 \\ 1 & -2 & -2 & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix}$ (i) $\begin{bmatrix} 2 & 0 & -1 & 1 \\ 1 & 1 & -1 & 0 \\ -1 & 0 & 6 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$