Systems of Equations, Cramer's Rule, Gaussian Elimination Method

Mathematics I — Exercises for Land Resources Management

Academic Year 2025/2026

Prof. UPP Agnieszka Łacka, PhD, DSc

Department of Mathematical and Statistical Methods

Poznań University of Life Sciences

1. Solve the systems of equations using the inverse matrix and Cramer's rule:

(a) $\begin{cases} 9x_1 - 8x_2 = 4 \\ 7x_1 + 2x_2 = 3 \end{cases}$

(b) $\begin{cases} x_1 + 7x_2 = 2 \\ 2x_1 - x_2 = 9 \end{cases}$

(c) $\begin{cases} x_1 + 2x_2 - x_3 = 1 \\ 3x_1 + x_2 + x_3 = 2 \\ x_1 - 5x_3 = 0 \end{cases}$

(d) $\begin{cases} x_1 - 2x_2 + 3x_3 = -7 \\ 3x_1 + x_2 + 4x_3 = 5 \\ 2x_1 + 5x_2 + x_3 = 18 \end{cases}$

(e) $\begin{cases} x_1 + 2x_2 + 3x_3 = 14 \\ 4x_1 + 3x_2 - x_3 = 7 \\ x_1 - x_2 + x_3 = 2 \end{cases}$

(f) $\begin{cases} x_2 + x_3 + x_4 = 4 \\ x_1 + x_3 + x_4 = -1 \\ x_1 + x_2 + x_4 = 2 \\ x_1 + x_2 + x_3 = -2 \end{cases}$

(g) $\begin{cases} 2x_1 + x_2 = 1 \\ 5x_1 - 2x_2 = 7 \end{cases}$

(h) $\begin{cases} 5x_1 - 2x_2 = 6 \\ 3x_1 + x_2 = 4 \end{cases}$

(i) $\begin{cases} 2x_1 + 3x_2 + x_3 = 7 \\ 3x_1 + x_2 + 2x_3 = 3 \\ 4x_1 - 2x_2 + x_3 = -1 \end{cases}$

(j) $\begin{cases} 3x_1 - 3x_2 + 2x_3 = 2 \\ 5x_1 - 6x_2 + 4x_3 = 3 \\ 4x_1 - 5x_2 + 2x_3 = 1 \end{cases}$

(k) $\begin{cases} x_1 + 2x_2 + 3x_3 - 4x_4 = 4 \\ x_2 - x_3 + x_4 = -3 \\ x_1 + 3x_2 - 3x_4 = 1 \\ -7x_2 + 3x_3 + x_4 = -3 \end{cases}$

(l) $\begin{cases} x + y + z + t = 1 \\ 2x + 2y + z + t = 0 \\ 3x + 2y + 3z + 2t = 3 \\ 6x + 4y + 3z + 2t = 2 \end{cases}$

Ans. (a) $\left[\frac{16}{37}, -\frac{1}{74}\right]^T$, (b) $\left[\frac{13}{3}, -\frac{1}{3}\right]^T$, (c) $\left[\frac{15}{28}, \frac{2}{7}, \frac{3}{28}\right]^T$, (d) $[2, 3, -1]^T$, (e) $[1, 2, 3]^T$, (f) $\left[-1\frac{2}{3}, 3\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]^T$, (g) $[1, -1]^T$, (h) $\left[\frac{14}{11}, \frac{2}{11}\right]^T$, (i) $[1, 2, -1]^T$, (j) $[1, 1, 1]^T$, (k) $[-8, 0, 0, -3]^T$, (l) $[1, -2, 0, 2]^T$

2. Solve the system of equations using the Gaussian elimination method:

(a) $\begin{cases} x_1 + 2x_2 + 3x_3 = 14 \\ 3x_1 + x_2 + 2x_3 = 11 \\ 2x_1 + 3x_2 + x_3 = 11 \end{cases}$

(b) $\begin{cases} 2x - 3y = 8 \\ x + y = -1 \\ 5x - y = 7 \end{cases}$

(c) $\begin{cases} -x_1 + 3x_2 + 2x_3 = -1 \\ 2x_1 - x_2 + x_3 = -3 \\ x_1 - x_3 = 2 \\ x_1 + 2x_2 + 3x_3 = -4 \end{cases}$

(d) $\begin{cases} x + y - 4z = 0 \\ 2x + 2y - 8z = 1 \\ 2x + 5y - 20z = -1 \end{cases}$

(e) $\begin{cases} x_1 + 3x_2 - 4x_3 = 4 \\ 3x_1 + 2x_2 - x_3 = 1 \\ x_1 - 4x_2 + 7x_3 = 5 \end{cases}$

(f) $\begin{cases} x + y + z = -1 \\ 2x - y + z = 2 \\ 5x - y + 3z = 3 \\ 7x - 2y + 4z = 5 \end{cases}$

(g) $\begin{cases} 2x_1 - x_2 + 3x_3 = 1 \\ -x_1 + x_2 - 2x_3 = 2 \\ x_1 + x_3 = 3 \end{cases}$

(h) $\begin{cases} x - 2y + z + t = 1 \\ x - 2y + z - t = -1 \\ x - 2y + z + 5t = 5 \end{cases}$

(i) $\begin{cases} x_1 - 2x_2 + 3x_3 = -7 \\ 3x_1 + x_2 + 4x_3 = 5 \\ 2x_1 + 5x_2 + x_3 = 18 \end{cases}$

(j) $\begin{cases} x + 5y = 2 \\ -3x + 6y = 15 \end{cases}$

(k) $\begin{cases} x - 2y + 3z = -7 \\ 3x + y + 4z = 5 \\ 2x + 5y + z = 18 \end{cases}$

(l) $\begin{cases} x_1 + 2x_2 - 3x_3 = 0 \\ 4x_1 + 8x_2 - 7x_3 + x_4 = 1 \\ x_1 + 2x_2 - x_3 + x_4 = 1 \\ -x_1 + x_2 + 4x_3 + 6x_4 = 0 \end{cases}$

(m) $\begin{cases} 2x_1 + x_2 - x_3 + x_4 = 1 \\ x_2 + 3x_3 - 3x_4 = 1 \\ x_1 + x_2 + x_3 - x_4 = 1 \end{cases}$

(n) $\begin{cases} x + 2y - z - t = 1 \\ x + y + z + 3t = 2 \\ 3x + 5y - z + t = 3 \end{cases}$

Ans. (a) $[1, 2, 3]^T$, $[1, -2]^T$, $[0, 1, -2]^T$; (b) inconsistent, inconsistent; (c) indeterminate, indeterminate, indeterminate (d) $[2, 3, -1]^T$, $[-3, 1]^T$; (e) $[2, 3, -1]^T$, $[4, -2, 0, 1]^T$; (f) indeterminate, inconsistent