Eigenvectors and Eigenvalues

Mathematics I — Exercises for Land Resources Management

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1. Using the definition, check whether the given vectors are eigenvectors of the indicated matrices:

(a) $\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 4 & 1 \\ -1 & 6 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

(d) $\mathbf{A} = \begin{bmatrix} 5 & 1 & 4 \\ 3 & 3 & 2 \\ 6 & 2 & 10 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$

(e) $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (*the vector is not an eigenvector of* $\mathbf{A}$)

(f) $\mathbf{A} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (*the vector is an eigenvector for eigenvalue* $\lambda = -2$)

2. Find the eigenvalues and eigenvectors of the given matrices:

(a) $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -4 \\ 5 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & -2 & 5 \\ 0 & 1 & 4 \\ 0 & -1 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 1 & 0 \\ -4 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 6 & 3 \\ -1 & 5 & 1 \\ -1 & 2 & 4 \end{bmatrix}$ (f) $\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$

(g) $\begin{bmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{bmatrix}$ (h) $\begin{bmatrix} -1 & -3 & -2 \\ -1 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$

Answer: (a) $\lambda_1 = 3 \to [1,1]^T$, $\lambda_2 = 0 \to [-2,1]^T$

(b) $\lambda_1 = 2 \to [1,5/4]^T$, $\lambda_2 = 3 \to [1,1]^T$

(c) $\lambda_1 = 3 \to [1,0,0]^T$

(d) $\lambda_1 = 0 \to [1,-2,0]^T$, $\lambda_2 = 3 \to [0,0,1]^T$

(e) $\lambda_1 = 3 \to [1,0,1]^T; [2,1,0]^T$

(f) $\lambda_1 = 1 \to [-1,1]^T$, $\lambda_2 = 5 \to [3,1]^T$

(g) $\lambda_1 = 2 \to [1,2,0]^T; [0,0,1]^T$

(h) $\lambda_1 = -2 \to [-1,-1,1]^T$, $\lambda_2 = 0 \to [1,-1,1]^T$, $\lambda_3 = 4 \to [-1,1,1]^T$

3. Show that the sum of the eigenvalues is equal to the trace of the matrix (for the matrices from Exercise 2).

4. Show that the product of the eigenvalues is equal to the determinant of the matrix (for the matrices from Exercise 2).